

	P.R.Government College (Autonomous) KAKINADA	Program&Semester II B.Sc Major & Minor (III Sem) w.e.f 2025-26 admitted batch			
Course Code MAT – 301 T	TITLE OF THE COURSE Group Theory & Problem Solving Sessions				
Teaching	Hours Allocated: 60 (Theory)	L	T	P	C
Pre-requisites:	Basic Mathematics Knowledge on sets and number system.	3	-	-	3

Course Objectives:

To provide the learner with the skills, knowledge and competencies to carry out their duties and responsibilities in pure Mathematic environment.

Course Outcomes:

On Completion of the course, the students will be able to-	
CO1	Acquire the basic knowledge and structure of groups
CO2	Get the significance of the notation of a subgroup and cosets.
CO3	Understand the concept of normal subgroups and properties of normal subgroup, permutation and cyclic groups.
CO4	Study the homomorphisms and isomorphisms with applications.

Course with focus on employability/entrepreneurship /Skill Development modules

Skill Development		Employability		Entrepreneurship	
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UNIT I :

GROUPS : Binary Operation – Algebraic structure – semi group-monoid – Group definition and elementary properties Finite and Infinite groups – examples – order of a group, Composition tables with examples.

UNIT II:

SUBGROUPS: Complex Definition – Multiplication of two complexes Inverse of a complex-Subgroup definition-examples-criterion for a complex to be a subgroups; Criterion for the product of two subgroups to be a subgroup-union and Intersection of subgroups. Coset

Definition – properties of Cosets – Index of a subgroups of a finite groups – Lagrange's Theorem.

UNIT III:

NORMAL SUBGROUPS:

Normal Subgroups: Definition of normal subgroup – proper and improper normal subgroup– Hamilton group- Criterion for a subgroup to be a normal subgroup – intersection of two normal subgroups Sub group of index 2 is a normal sub group.

UNIT IV

HOMOMORPHISM:

Quotient groups, Definition of homomorphism – Image of homomorphism elementary properties of homomorphism – Isomorphism – automorphism definitions and elementary properties–kernel of a homomorphism – fundamental theorem on Homomorphism and applications.

UNIT V:

PERMUTATIONS AND CYCLIC GROUPS:

Definition of permutation – permutation multiplication – Inverse of a permutation – cyclic permutations – transposition – even and odd permutations – Cayley's theorem.

Cyclic Groups - Definition of cyclic group – elementary properties – classification of cyclic groups.

Co-Curricular Activities

Seminar/ Quiz/ Assignments/ Group theory and its applications / Problem Solving.

TEXT BOOK

Modern Algebra by A.R.Vasishtha and A.K.Vasishtha, KrishnaPrakashanMedia Pvt. Ltd., Meerut.

REFERENCE BOOKS :

1. Abstract Algebra by J.B. Fraleigh, Published by Narosa publishing house.
2. Modern Algebra by M.L. Khanna, Jai Prakash and Co. Printing Press, Meerut
3. Rings and Linear Algebra by Pundir&Pundir, published by PragathiPrakashan

BLUE PRINT FOR QUESTION PAPER PATTERN

SEMESTER-III

Unit	TOPIC	S.A.Q	E.Q	Marks allotted to the Unit
I	Groups	2	1	20
II	Subgroups , Co-sets and Lagrange's Theorem	1	2	25
III	Normal subgroups	1	1	15
IV	Homomorphism	1	1	15
V	Permutations and Cyclic Groups	2	1	20
Total		7	6	95

S.A.Q. = Short answer questions (5 marks)

E.Q = Essay questions (10 marks)

Short answer questions : 4 X 5 = 20 M

Essay questions : 3 X 10 = 30 M

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Total Marks = 50 M

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Pithapur Rajah's Government College (Autonomous), Kakinada
II year B.Sc., Degree Examinations - III Semester
Course: Major V & Minor II : Group Theory & Problem Solving Session
Model Paper(w.e.f. 2025-26)

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Time: 2Hrs

Max. Marks: 50

SECTION-A

Answer any three questions. Selecting at least one question from each part

Part – I

3 X 10 = 30

1. Essay question from Unit -I.
2. Essay question from Unit – II.
3. Essay question from Unit – II.

Part – II

4. Essay question from Unit – III.
5. Essay question from Unit – IV.
6. Essay question from Unit - V.

SECTION-B

Answer any four questions

4 X 5 M = 20 M

7. Short answer question from Unit - I.
8. Short answer question from Unit – I.
9. Short answer question from Unit – II.
10. Short answer question from Unit - III.
11. Short answer question from Unit – IV.
12. Short answer question from Unit -V.
13. Short answer question from Unit - V.

PITHAPUR RAJAH'S GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
DEPARTMENT OF MATHEMATICS

Question Bank

PAPER–Major V & Minor II : Group Theory & Problem Solving Session

Short Answer Questions

UNIT-1

1. Prove that in a group the identity element is unique and the inverse of every element is unique.
2. If G is a group, for $a, b \in G$ prove that $(ab)^{-1} = b^{-1} a^{-1}$
3. If every element of a group G is its own inverse, prove that G is abelian.
4. Prove that in a group $G(\neq \emptyset)$, for $a, b, x, y \in G$, the equations $ax = b, ya = b, \forall a, b \in G$ have unique solutions.
5. Prove that the group $(G, \bullet)$ is abelian iff $(ab)^2 = a^2 b^2$

UNIT-II

6. If a non -empty complex H of a group G is a subgroup of G then prove that $H = H^{-1}$.
7. If H is a subgroup of a group G then show that $HH = H$.
8. If H and K are two subgroups of a group G then show that $H \cap K$ is also a subgroup of G .
9. Let h be a subgroup of a group of G and $a, b \in G$ then prove that $Ha = Hb$ iff $ab^{-1} \in H$.

UNIT-III

10. Prove that every subgroup of an abelian group is normal.
11. Show that a subgroup H of a group G is normal iff $xHx^{-1} = H$ for all $x \in G$
12. Prove that intersection of two normal sub-groups of a group is a normal sub-group.
13. Prove that a subgroup of index 2 in a group is a normal subgroup.

UNIT-IV

14. Prove that every quotient group of an abelian group is abelian.
15. If f is a homomorphism of a group G into a group G_1 then show that the kernel of f is a normal subgroup of G .
16. Prove that every homomorphic image of an abelian group is abelian.
17. Let G be a group. If $f : G \rightarrow g$ defined by $f(x) = x^2$ for all $x \in G$ is a homomorphism then show that G is abelian.

UNIT-V

18. Express the permutation $(1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9)$ as a product of disjoint cycles.
19. Show that the permutation $(1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9)$ is odd permutation.
20. Verify whether the permutation $(1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9)$ is even or odd.
21. Prove that every cyclic group is an abelian group.
22. Prove that a group of prime order is cyclic.
23. Prove that if G is an infinite cyclic group, then G has exactly two generators

Essay Questions

UNIT-1

1. Show that the set Q^+ of all positive rational numbers forms an abelian group under the composition defined by ' $\circ$ ' such that $a \circ b = (ab)/3$ for $a, b \in Q^+$.

2. Prove that the set G of rational numbers other than 1 with operation $*$ such that $a * b = a + b - ab$ for all $a, b \in G$ is an abelian group.
3. Prove that the set of n th roots of unity forms an abelian group w.r.t. ' $\cdot$ '
4. Prove that a finite semi-group $(G, \cdot)$ satisfying the cancelation laws is a group.

UNIT-II

5. Prove that a non-empty complex H of a group G is a subgroup of G if and only if $a, b \in H \Rightarrow ab^{-1} \in H$, where b^{-1} is the inverse of b in G .
6. If H and K are two sub-groups of a group G , then $H \cup K$ is a subgroup iff either $H \subseteq K$ or $K \subseteq H$.
7. If H and K are two sub-groups of a group G , then show that HK is a sub-group of G if and only if $HK = KH$.
8. State and prove Lagrange's Theorem. Prove that the converse of Lagrange's theorem is not true.
9. Prove that any two left cosets of a subgroup are either disjoint or identical.
10. If H is a subgroup of a group G , then there is one to one corresponding between the set of all distinct left cosets of H in G and the set of all distinct right cosets of H in G .

UNIT-III

11. Prove that H of a group G is normal sub-group of G if and only if each left coset of H in G is a right coset of H in G .
12. Prove that H is a normal sub-group of G if and only if product of two right (left) cosets of H in G is again a right (left) coset of H on G .
13. State and prove Fundamental theorem of homomorphism of groups.
14. If M, N are two normal subgroups of a group G such that $M \cap N = \{e\}$, then show that every element of M commutes with every element of N .

UNIT-IV

15. Prove that the set G/H of all cosets of a normal subgroup H in a group G with respect to coset multiplication is a group.
16. State and prove fundamental theorem of homomorphism of groups.
17. Show that the necessary and sufficient condition for a homomorphism f of a group G onto a group G^1 with kernel K to be an isomorphism of G into G^1 is that $K = \{e\}$.

UNIT-V

18. Let S_n be a symmetric group of n symbols and let A_n be the group of even permutations, then show that A_n is a normal in S_n and $O(A_n) = n! / 2$.
19. State and prove Cayley's theorem.
20. Prove that every subgroup of a cyclic group is cyclic.
21. Prove that the order of a cyclic group is equal to the order of its generators.
22. Prove that a cyclic group of order n has $\phi(n)$ generators.

	P.R.Government College (Autonomous) KAKINADA	Program&Semester II B.Sc Major & Minor (III Sem) w.e.f 2025-26 admitted batch			
Course Code MAT-301P	TITLE OF THE COURSE Group Theory & Problem Solving Sessions Practical Course				
Teaching	Hours Allocated: 30 (Practicals)	L	T	P	C
Pre-requisites:	Basic Mathematics Knowledge on sets and number system.	-	-	2	1

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GROUPS : Binary Operation – Algebraic structure – semi group-monoid – Group definition and elementary properties Finite and Infinite groups – examples – order of a group, Composition tables with examples.

UNIT II:

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HOMOMORPHISM:

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UNIT V:

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Semester – III End Practical Examinations

Scheme of Valuation for Practical's

Time : 2 Hours

Max.Marks : 50

- **Record** - 10 Marks
- **Viva voce** - 10 Marks
- **Test** - 30 Marks
- **Answer any 5 questions. At least 2 questions from each section. Each question carries 6 marks.**

BLUE PRINT FOR PRACTICAL PAPER PATTERN

**COURSE-MAJOR V & MINOR II – GROUP THEORY& PROBLEM SOLVING
SESSION**

Unit	TOPIC	E.Q	Marks allotted to the Unit
I	Groups	1	06
II	Sub groups Co-sets and Lagrange's Theorem	2	12
III	Normal subgroups	2	12
IV	Homomorphism	1	06
V	Permutations and Cyclic Groups	2	12
	Total	08	48

PITHAPUR RAJAH'S GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA

II year B.Sc., Degree Examinations - III Semester

Course-Major V & Minor II: Group Theory & Problem Solving Session

(w.e.f. 2024-25 Admitted Batch)

Practical Model Paper (w.e.f. 2025-2026)

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Time: 2Hrs

Max. Marks: 50M

Answer any 5 questions. At least 2 questions from each section.

5 x 6 = 30 Marks

SECTION - A

1. Unit - I.
2. Unit – II.
3. Unit - II.
4. Unit – III.

SECTION – B

5. Unit - III.
6. Unit - IV.
7. Unit – V.
8. Unit - V.

➤ **Record** - 10 Marks

➤ **Viva voce** - 10 Marks